Improving communication performance in dense linear algebra via topology-aware collectives

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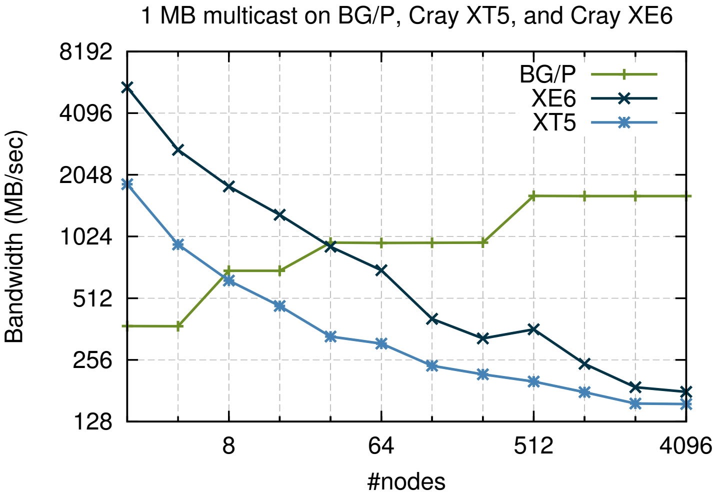
Outline

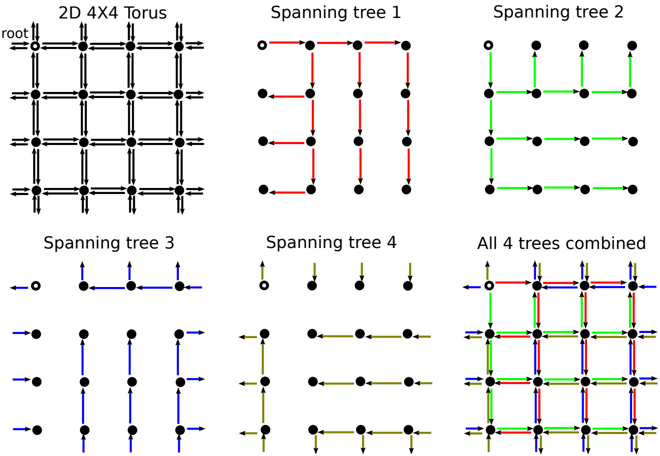
Performance of multicast (BG/P vs Cray)



Why the performance discrepancy in multicasts?

- Cray machines use **binomial multicasts**
  - Form spanning tree from a list of nodes
  - Route copies of message down each branch
  - Network contention degrades utilization on a 3D torus
- BG/P uses **rectangular multicasts**
  - Require network topology to be a k-ary n-cube
  - Form $2n$ edge-disjoint spanning trees
    - Route in different dimensional order
    - Use both directions of bidirectional network

2D rectangular multicasts trees



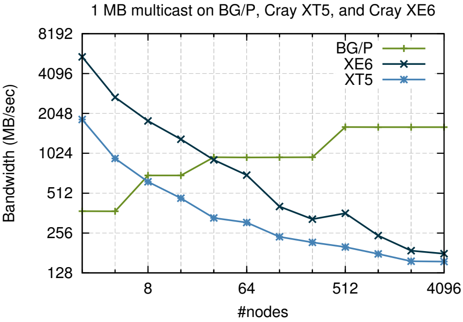
[Watts and Van De Geijn 95]

Another look at that first plot

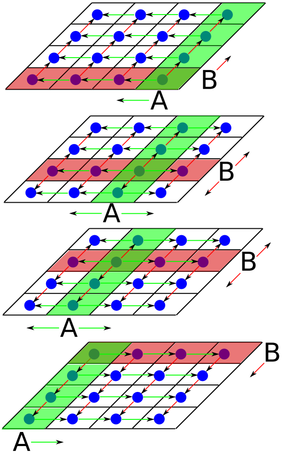
How much better are rectangular algorithms on $P = 4096$ nodes?

- Binomial collectives on XE6
  - **1/30th of link bandwidth**
- Rectangular collectives on BG/P
  - **4X the link bandwidth**
- **120X improvement in efficiency!**

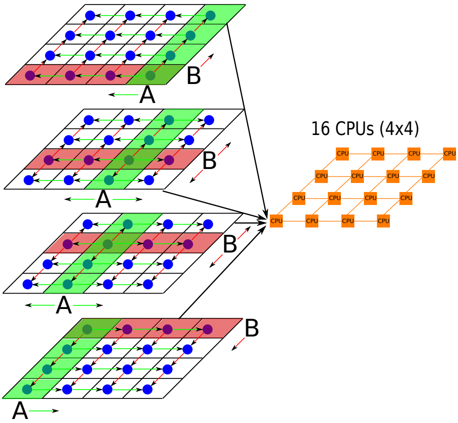
How can we apply this?



Matrix multiplication

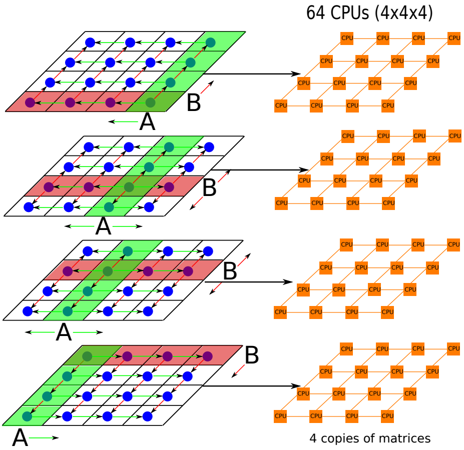


2D matrix multiplication



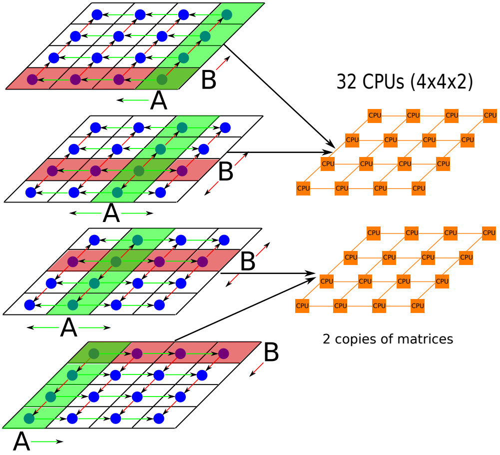
[Cannon 69], [Van De Geijn and Watts 97]

3D matrix multiplication

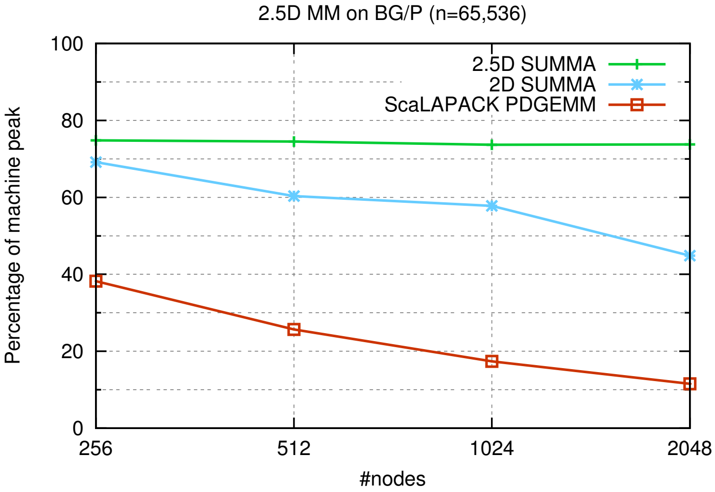


[Agarwal et al 95], [Aggarwal, Chandra, and Snir 90], [Bernsten 89]

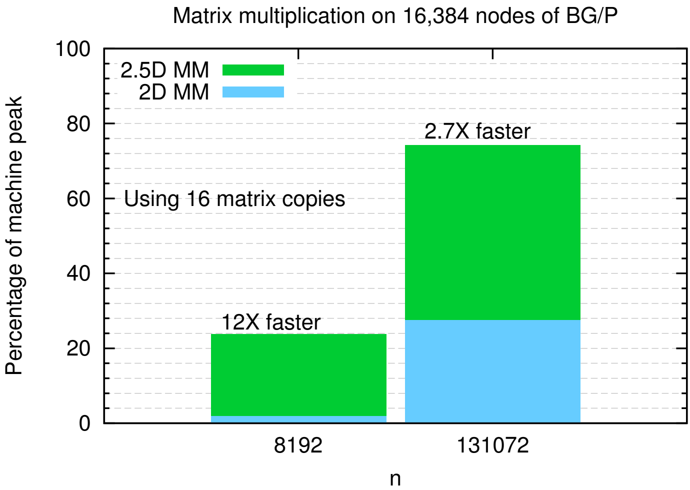
2.5D matrix multiplication



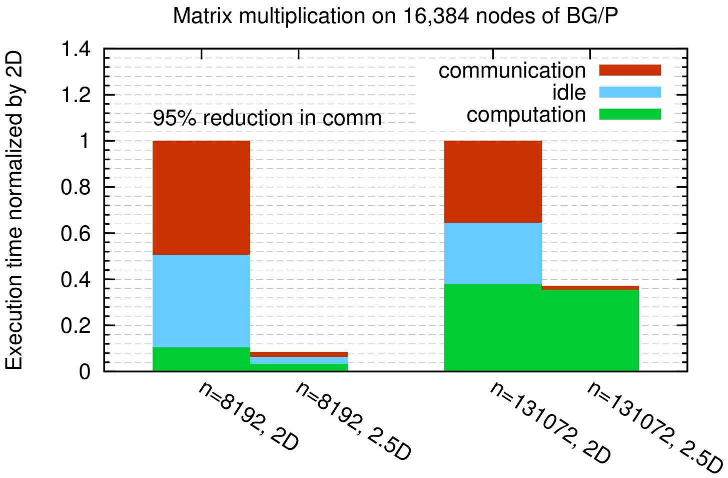
Strong scaling matrix multiplication



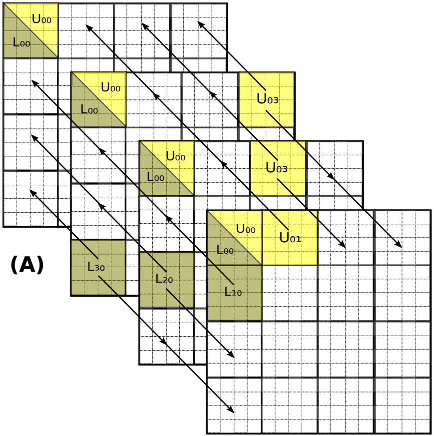
2.5D MM on 65,536 cores



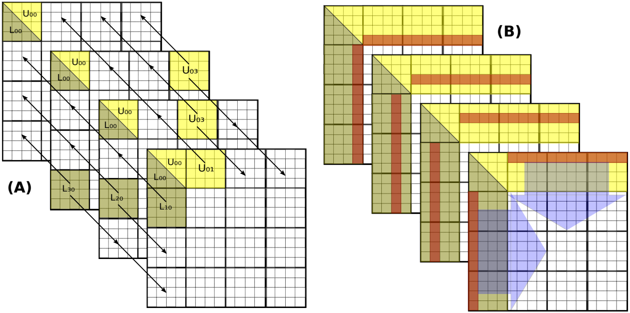
Cost breakdown of MM on 65,536 cores



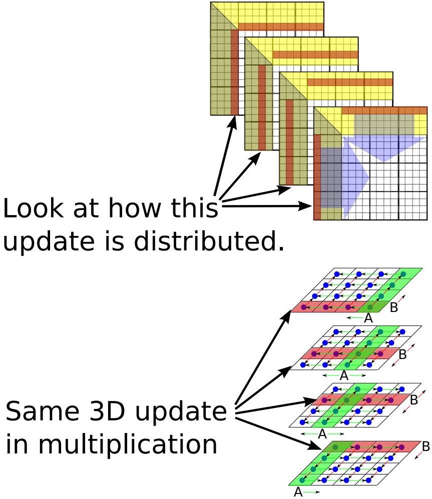
2.5D LU factorization



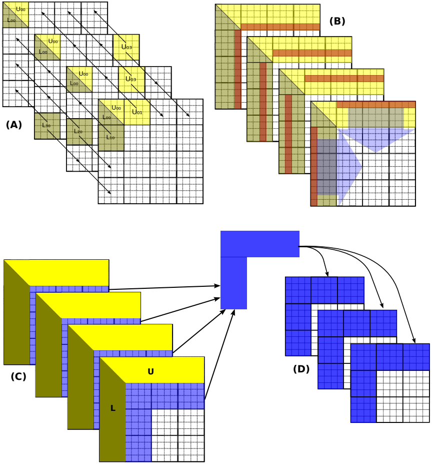
2.5D LU factorization



2.5D LU factorization

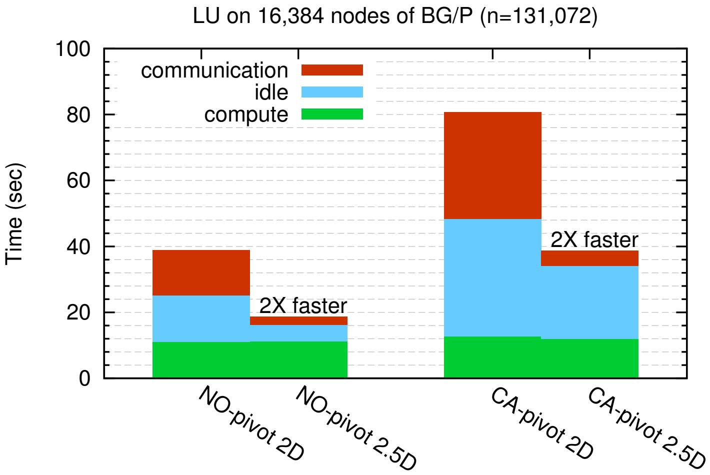


2.5D LU factorization

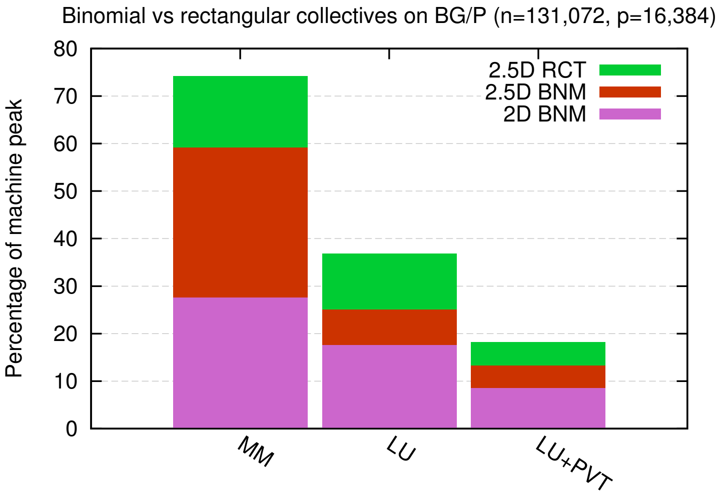


[Solomonik and Demmel, EuroPar '11, Distinguished Paper]

2.5D LU on 65,536 cores



Rectangular (RCT) vs binomial (BNM) collectives



A model for rectangular multicasts

$$t_{mcast} = m/B_n + 2(d+1)\cdot o + 3L + d\cdot P^{1/d}\cdot(2o+L)$$

Our multicast model consists of 3 terms

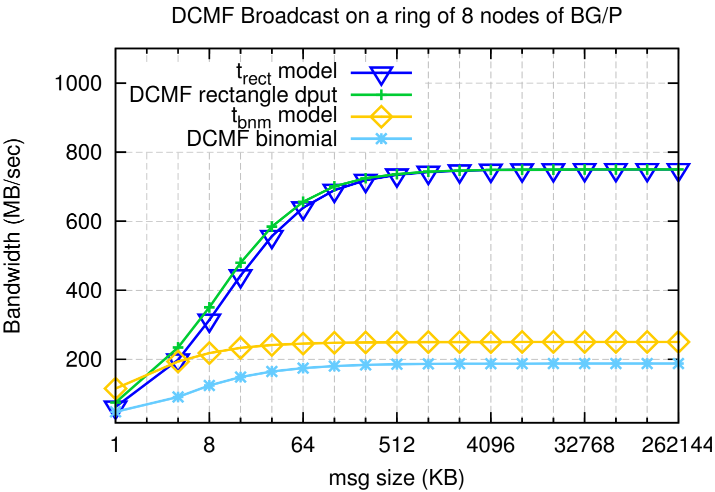
1. m/B_n, the bandwidth cost
2. $2(d+1)\cdot o + 3L$, the multicast start-up overhead
3. $d\cdot P^{1/d}\cdot(2o+L)$, the path overhead

A model for binomial multicasts

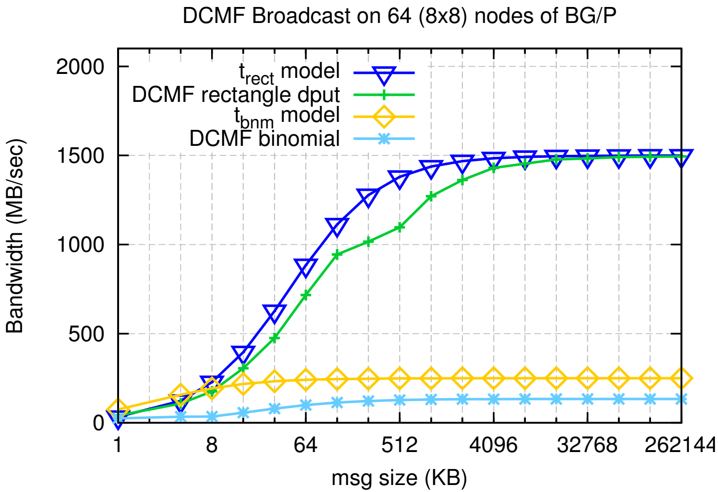
$$t_{bnm} = \log_2(P) \cdot (m/B_n + 2o + L)$$

- The root of the binomial tree sends $\log_2(P)$ copies of message
- The setup overhead is overlapped with the path overhead
- **We assume no contention**

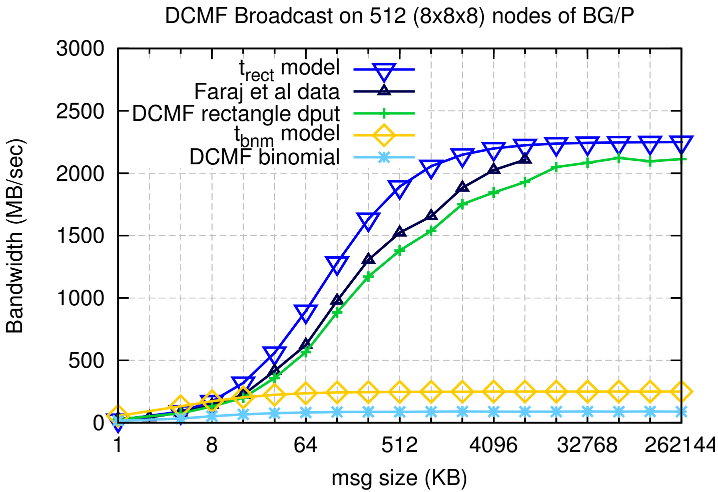
Model verification: one dimension



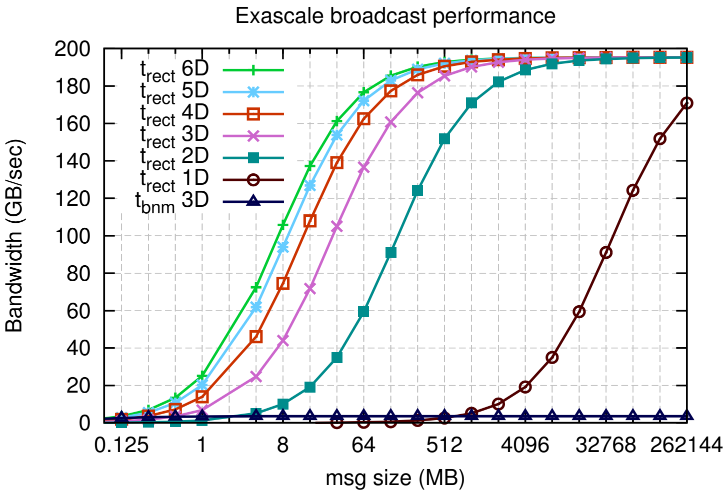
Model verification: two dimensions



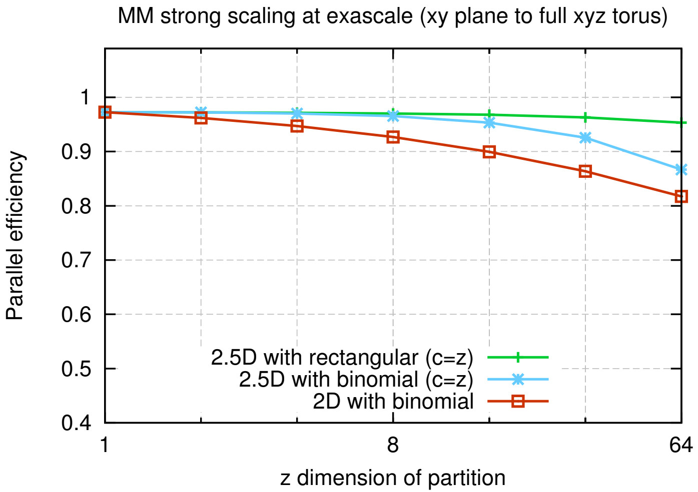
Model verification: three dimensions



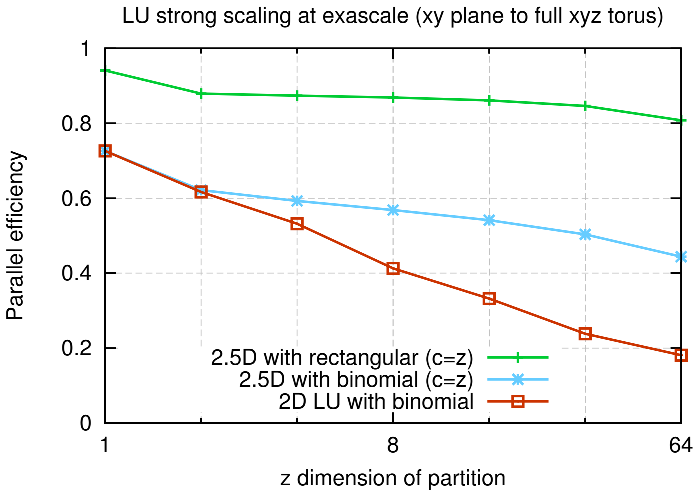
Modelling collectives at exascale ($p = 262,144$)



Modelling matrix multiplication at exascale



Modelling LU factorization at exascale



Conclusion

- Topology-aware scheduling
  - Present in IBM BG but not in Cray supercomputers
  - Avoids network contention/congestion
  - Enables optimized communication collectives
  - Leads to simple communication performance models
- Future work
  - An automated framework for topology-aware mapping
  - Tensor computations mapping
  - Better models for network contention

Acknowledgements

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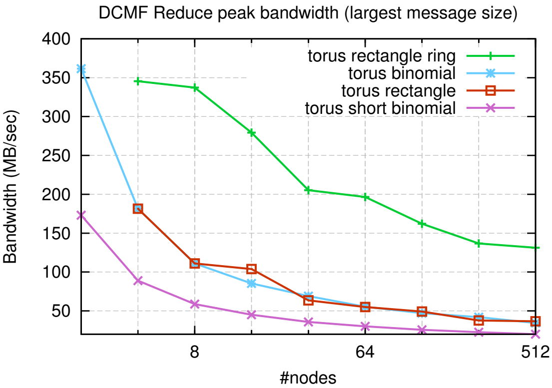
Backup slides

A model for rectangular reductions

$$t_{red} = \max[m/(8\gamma), 3m/\beta, m/B_n] + 2(d+1)\cdot o + 3L + d\cdot P^{1/d}\cdot(2o+L)$$

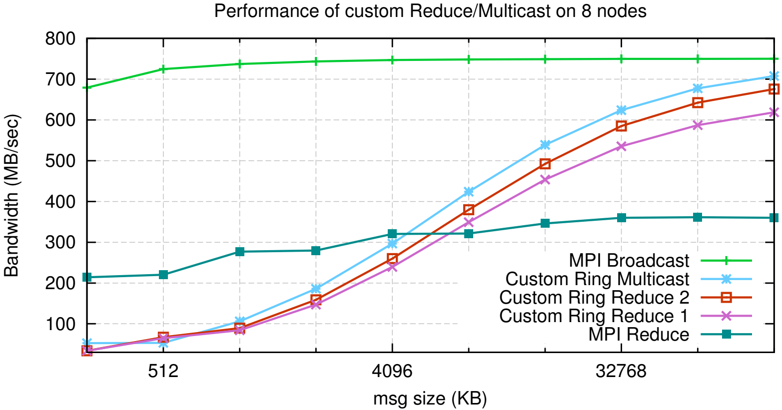
- Any multicast tree can be inverted to produce a reduction tree
- The reduction operator must be applied at each node
  - each node operates on $2m$ data
  - both the memory bandwidth and computation cost can be overlapped

Rectangular reduction performance on BG/P

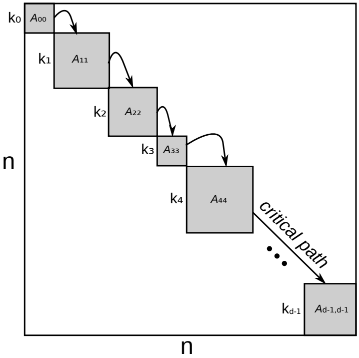


BG/P rectangular reduction performs significantly worse than multicast

Performance of custom line reduction

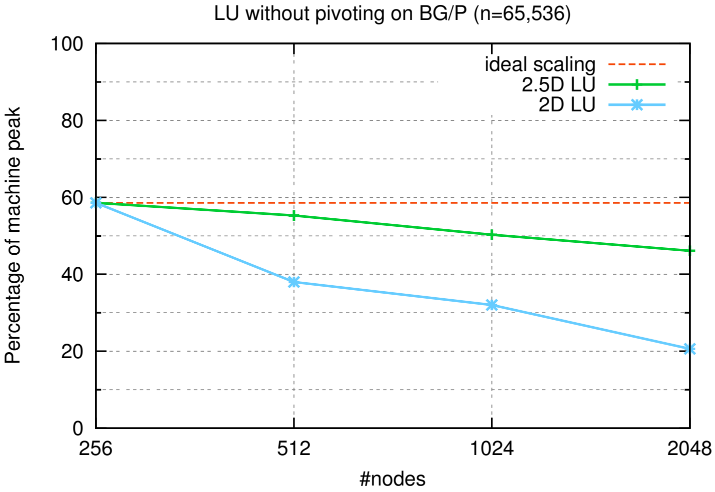


A new LU latency lower bound



flops lower bound requires $d = \Omega(\sqrt{p})$ blocks/messages
bandwidth lower bound required $d = \Omega(\sqrt{cp})$ blocks/messages

2.5D LU strong scaling (without pivoting)



Strong scaling of 2.5D LU with tournament pivoting